ISSN **2321-743X**

International Journal of Research in

Engineering and Bioscience

Volume **7** Issue **2** (Pages 1-4)

IJREB

Journal home page: www.ijreb.org

STUDY ON PROPERTIES OF ORDER DIVISOR GRAPHS OF FINITE GROUPS

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ABSTRACT

Exploring synergy between Algebra and Graph Theory, Algebraic Graph Theory delves into the intriguing realm where graphs are defined by the group G as their vertex set. This paper intricately examines the relationships between vertices a and b, each possessing unique orders. Adjacency between two vertices, a and b, is established when the order of one divides the order of the other, or vice versa. Within these discussions, various properties of the order divisor graphs are thoroughly analyzed.

Keywords: *- order divisor graph, Eulerian.*

INTRODUCTION

Algebraic graph theory constitutes a branch of mathematics where algebraic techniques are employed to tackle problems concerning graphs. Within this realm, the properties of graphs are scrutinized by transforming them into algebraic constructs, leveraging the tools and methodologies of algebra to derive theorems about these graphs. Furthermore, many algebraic structures can be explored through the properties of graphs and by representing them graphically.

One branch of algebraic graph theory revolves around examining graphs in conjunction with linear algebra. Various matrices associated with a graph serve as prevalent algebraic structures, aiding in characterizing the graph effectively. Another branch encompasses the exploration of symmetry and regularity properties of groups, employing group theory. A graphical portrayal of a group can be achieved through a set of generators and relations, linking graph theory and group theory, thus offering a visual approach to understanding groups while bridging two significant branches of mathematics, Algebra and Graph Theory.

Groups serve as the fundamental mathematical instruments for investigating the symmetries of objects, with these symmetries often intertwined with graph automorphisms. Many abstract algebraic structures find their roots as special instances of groups. Group theory stands as a prominent field in mathematics due to its wide-ranging applications across various domains such as biochemistry, electricaleng., com.science, and operationsresearch. Both branches of mathematics, group theory, and graph theory, play indispensable roles in modern mathematical pursuits. Group theory entails the study and analysis of diverse groups structures, while graph theory focuses on representing the structures of materials and objects through graphs.

1. Preliminaries

Definitions

A graph $G = (V, E)$ comprises a set of objects $V = \{v_1, v_2, ..., v_n\}$, termed vertices, and another set $E = \{e_1, e_2, ..., e_n\}$ whose elements are edges, with each edge associated with an unordered pair (v_i, v_j) of vertices. An Eulerian path traverses every edge exactly once in a finite graph, permitting revisits to vertices. A planar graph is one that can be depicted on a plane, ensuring edges intersect solely at their endpoints. A complete k-partite graph connects every pair of vertices from distinct independent sets in a k-partition. Furthermore, a complete multipartite graph is one that is fully k-partite for some k.

Let Hbeaset equipped with binary operation assigning to each ordered pair (a,b) of its elements an element in$H$. We define H as a group under this operation if it satisfies three properties: Associativity, Identity element and Inverse element. The quantity of elements within a group, whether finite or infinite, is referred to as its order, denoted by $|H|$. The order of an element g in a group H is the smallest positive integer n such that ng=ng=e, in additive notation. The group of symmetries of a regular polygon with nn sides (where $n \geq 3$) is termed a dihedral group of order n, denoted by D_n. The group of units of Z_n, denoted by U_n={$x \in Z: 1 \leq x < n$ and $(x, n) = 1$}. Let G be a finite group. Then $\Psi(H)$ denotes the ***order divisor graph*** whose vertex set is H suchthat there exist an edge between two distinct vertices a and b having different orders if $o(a) \mid o(b)$ or $o(b) \mid o(a)$.

Theorems used

Theorem 1.1 [7]:

The $\Psi(H)$ of the dihedral group $D_m(m \geq 3)$ is a star graph iff if m is prime.

Theorem 1.2 [7]:

Let H be a finite of order p^n. $\Psi(H)$ is a multi partite graph which is also a complete graph.

Theorem 1.3 [3]:

A graph is nonplanar iff it has a subgraph holomorphic to either K_5 or $K_{3,3}$.

Theorem 1.4 [6]:

The degree of each vertex of a simple graph is even if and only it is Eulerian.

Theorem 1.5 [8]:

The multiplicative group U_n is cyclic if and only if n is either $1,2,4, p^k$ or $2p^k$ for some odd prime p and a non negative integer k.

Result 1.6 [4]:

In a finite cyclic group, the order of an element divides the order of the group.

2. More Results on Order divisor graphs

Theorem 1: Let H bea group with order $p^n(\neq 2)$ then $\Psi(H)$ is non- Eulerian.

Proof: Let H be a finite group with order $p^n(\neq 2)$.Assume that $\Psi(H)$is Eulerian. Then by theorem 1.4, each vertex has an even degree. From theorem 1.2, $\Psi(H)$ is a multi-partite graph which is also complete. Hence, the degree of each vertex is ≥ 2, indicating that the degree of the vertex can either be even or odd. This contradicts Theorem 1.4. Therefore, by contraposition, the result follows.

Corollary 2: $\Psi(H)$ of the Dihedral group $D_n(n \geq 3)$, is planar iff if $n$isprime.

Proof: By theorem 1.1, $\Psi(H)$ of the dihedral group $D_n(n \geq 3)$ is a star graph iff n is prime. A star graph is clearly planar by theorem 1.3.

Theorem 2.5: If $H = U_n$ and $n = p^k, 2p^k, p$ an odd primeand $k \geq 2$,then $\Psi(H)$ is not a star graph.

Proof: Let $H = U_n$ and $n = p^k, 2p^k, p$ an odd $prime$ and $k \geq 2$ .We know that U_n={$x \in Z: 1 \leq x < n$ and $(x, n) = 1$}. This refers to the group of units in the ring Z_n, which comprises $\phi(n)$ elements, where ϕ represents Euler's totient function. By theorem 1.5, this group is cyclic. That is, if $n = 1,2,4, p^k, 2p^k$, p an odd prime, then there exists atleast one generator, say a. By Lagrange's theorem, we vunderstand that the order of a always divides (n). In this context, the order of a is indeed equal to (n), making it as large as possible. By result 1.6, the order of an element divides the order of the group. That is, we can say that order of atleast one element divide order of another. Thus we can conclude that order divisor graph of U_n where $n = p^k, 2p^k, p$ an odd prime and $k \geq 2$ is not a star graph.

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